ASYMPTOTIC THEORY OF CHEMICALLY UNSTABLE LAYER CLOSE TO A PERFECTLY CATALYTIC WALL*

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The statement and asymptotic solution of the problem of ionization equilibrium deviation near a perfectly catalytic wall in a weakly ionized gas is considered.Ratios of the unperturbed recombination length to the gas temperature variation scale and of the characteristic temperature of gas to the ionization potential are taken as the small parameters. The nonequilibrium layer is analyzed in detail using the method of merging asymptotic expansions in a small parameter. The physical meaning of obtained results is discussed.

The investigation of chemically unbalanced flows of gas was the subject of numerous publications in connection with the determination of volt-ampere characteristics of the boundary layer of electrodes and, also, in connection with problems of external aerodynamics (see, e.g. /1-4/). The majority of published papers contain mainly the results of numerical calculations. There are also papers dealing with the asymptotic theory of nonequilibrium flows at low Damkeler numbers /5,6/. The asymptotic theory is considered here in the opposite limit case of high Damkeler numbers, when the flow of gas in the outer part of the gasdynamic boundary layer is in chemical equilibrium.

1. Statement of the problem. Let us consider the following simple model. A plane perfectly catalytic surface borders on a weakly ionized gas containing a neutral basic component, atoms and ions of an easily ionized additive, and electrons. The ionization of the additive atoms is effected by an electron impact; at recombination the third body is the electron. The condition of quasineutrality is assumed satisfied throughout the volume of gas. For simplicity of exposition we assume that the gas is, as a whole, quiescent and that the temperature distribution is defined by a fairly smooth function of the y-coordinate (the y-axis is normal to the wall); as the distance from the wall increases, the gas temperature approaches a constant value.

For the determination of the quasineutral molar concentration of charged particiles x we have on the above assumptions the following nonlinear boundary value problem /1,7/:

$$\frac{d}{dy}\left(nD\frac{dx}{dy}\right) = k_r n^3 x \left(x^2 - x_r^2\right); \quad x = \frac{n_r}{n}, \quad x_r = \frac{n_{er}}{n}$$

$$y = 0, \ x = 0; \quad y \to \infty, \ x \to x_{t\infty}$$
(1.1)

where n_e is the quasineutral concentration of charged particles, n is the over-all concentration of gas particles, n_{er} is the local chemically stable quasineutral concentration of charged particles, D is the coefficient of ambipolar diffusion, and k_r is the constant of the recombination rate. The subscript ∞ denotes here and below quantities at some distance from the wall.

We pass to normalized variables $\eta = y/L$ and $z = x/x_{r\infty}$ (L is the characteristic scale of gas temperature variation), and rewrite problem (1.1) in the form:

$$\chi (az')' = bz (z^2 - r^2)$$

$$\eta = 0, z = 0; \eta \to \infty, z \to 1$$

$$\chi = \frac{d_{\infty}}{2L^2}, \quad d = \left(\frac{2D}{k_r n_{er}^2}\right)^{1/2}, \quad a = \frac{nD}{n_{\infty} D_{\infty}}, \quad b = \frac{k_r n^3}{k_{r\infty} n_{\infty}^3}, \quad r = \frac{x_r}{x_{r\infty}}$$

$$(1.2)$$

$$(1.3)$$

where the prime indicates differentiation with respect to η , and the quantity d has the meaning of the local recombination length.

Assuming for simplicity that the gas presssure and the molar concentration of atoms of the additive are constant, for function r in conformity with the Saha equation we have

$$r = \theta^{*/} e^{-t}, \quad \theta = \frac{T}{T_{\infty}}, \quad t = mq, \quad m = \frac{I}{2kT_{\infty}}, \quad q = \frac{1-\theta}{\theta}$$

where T is the gas temperature, k is the Boltzmann constant, and I is the ionization potential of atoms of the additive.

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The dimensionless temperature θ of gas is assumed to be specified by a monotonically increasing function of the η -coordinate. The quantities a and b are known functions of the gas temperature also specified by functions of the η -coordinate. The problem (1.2), (1.3) thus contains as coefficients the specified functions $a(\eta), b(\eta)$, and $\theta(\eta)$, and the specified parameters χ and m.

2. The asymptotic formulation. In the particular case of constant gas temperature (i.e. without allowance for the wall cooling effect) $a \equiv b \equiv r \equiv 1$ the problem (1.2), (1.3) has an exact analytic solution /1,8/, which does not exist in the general case of variable temperature.

In many practically interesting cases parameter χ is small, and the problem (1.2), (1.3) may be considered to belong to the type of singularly perturbed (e.g., when the characteristic conditions in a MGD generator are $T_{\infty} = 2700$ K, L = 1 cm, pressure 1 atm, molar concentration of the additive (atoms of potassium) 1%, and $D_{\infty} = 1 \text{ cm}^2/\text{s}$ we have $\chi = 3.4 \cdot 10^{-6}$ and m = 9.3). An effective method of analyzing problems of this type is that of external and internal asymptotic expansions in a small parameter /9/. The first term of the asymptotic expansion of problem (1.2), (1.3) solution is the well known solution that corresponds to chemical equilibrium throughout the layer. The internal problem was, as far as the authors are aware, not previously considered in literature.

Asymptotic formulation of the considered problem essentially depends on the wall temperature. In the case of a "hot" wall whose temperature is so high that the corresponding molar concentration of charged particles is of the same order of magnitude as that of unperturbed concentration far away from the wall (i.e. $r_w = O(1)$, where the subscript w denotes quantities at the wall), parameter χ is the single small parameter of the problem. In the case of a "cold" wall, when $r_w \ll 1$ (for instance, if $T_w = 800$ K is assumed in the indicated above conditions, we have $r_w = 5.6 \cdot 10^{-11}$), it is necessary to introduce besides parameter one more small parameter related to r_w in the asymptotic formulation of the problem. The most natural is to use the parameter m^{-1} as the small parameter. Note that a small parameter of this type is basic in the combustion theory /10/.

The analysis of the two indicated limit cases enables us to investigate the whole range of the wall temperature variation.

3. The hot wall limit $(\chi \to 0)$. We seek an external asymptotic expansion of the solution of problem (1.2), (1.3) of the form

$$z(\eta; \chi) = z_1(\eta) + \dots$$
 (3.1)

whose substitution into Eq. (1.2) yields $z_1 = r(\eta)$. The internal expansion is of the form

 $z(\eta; \chi) = z_2(\eta_2) + \ldots; \quad \eta_2 = \eta / \sqrt{\chi}$

Its substitution into Eq. (1.2) yields

$$a_w d^2 z_2 / d\eta_2^2 = b_w z_2 \left(z_2^2 - r_w^2 \right) \tag{3.2}$$

for which the boundary condition at the wall is the same as the first of conditions (1.3) and the boundary condition at infinity is the following condition of merging with the external expansion (3.1):

$$\eta_2 = 0, \quad z_2 = 0; \quad \eta_2 \to \infty, \, z_2 \to r_w \tag{3.3}$$

The solution of the problem (3.2), (3.3) in the initial variables, has the form /1,8/

$$\boldsymbol{x} = \boldsymbol{x}_{vw} \text{ th } (\boldsymbol{y} / \boldsymbol{d}_w) \tag{3.4}$$

The governing factor for the calculation of volt-ampere characteristics of electric probes in plasma and of electrodes is the magnitude of the derivative of the quasineutral molar concentration of charged particles /7,11/. Differentiating (3.4) we obtain

$$(3.5)$$

4. The cold wall limit $(\chi \to 0, m^{-1} \to 0)$. Depending on the relation between the orders of magnitude of the small parameters χ and m^{-1} we have three limit cases: $m / \ln \gamma^{-1} \to k_1, m / \ln \gamma^{-1} \to 0, m / \ln \gamma^{-1} \to \infty$

where k_1 is some specified positive constant, and the small parameter $\gamma = \gamma(\chi)$ is related to parameter χ as follows:

$$\gamma = \chi^{1/2} \ln \gamma^{-1}$$

The solution of this nonlinear algebraic equation is of the form

$$\gamma = \chi^{1/2} \ln \chi^{-1/2} \left[1 - O\left(\frac{\ln \ln \chi^{-1/2}}{\ln \chi^{-1/2}}\right) \right]$$

4.1. $m/\ln \gamma^{-1} \rightarrow k_1$. The external expansion can be calculated directly

$$z(\eta; \chi, m) = r \left\{ 1 + \frac{a}{2b\theta^{4/z}} (k_1 q')^2 \exp\left[-2(\ln \gamma^{-1} - mq)\right] + \dots \right\}$$

$$r = r(\eta; m) = \theta^{4/z} e^{-mq}$$
(4.1)

The region of this expansion applicability is evidently bounded by the condition

$$\ln \gamma^{-1} - mq \to \infty$$

since its nonfulfillment violates the assumption that the second term of expansion (4.1) is small as compared to the first, an assumption that is basic in the derivation of that expansion. This shows that the structure of solution of this problem substantially depends on the value of k_1q_w . When $k_1q_w < 1$ condition (4.2) is satisfied for all $\eta \ge 0$; expansion (4.1) is applicable for $\eta > 0$. When $k_1q_w = 1$ condition (4.2) is satisfied for $\eta > 0$ and expansion (4.1) is applicable for $\eta > 0$. In the case of $k_1q_w > 1$ condition (4.2) is satisfied for $\eta > 0$ and expansion (4.1) is applicable for $\eta > 0$. In the solution of equation $k_1q(\eta_s) = 1$ and expansion (4.1) is applicable for $\eta > \eta_s$.

4.1.1. $k_1q_w < 1$. The internal expansion is of the form

$$z(\eta; \chi, m) = \exp(-mq_w) z_3(\eta_3)^+...; \eta_3 = \eta \ln \gamma^{-1} \exp(\ln \gamma^{-1} - mq_w)$$

For function z_3 we obtain the boundary value problem similar to problem (3.2), (3.3). In input variables we again obtain formulas (3.4) and (3.5).

4.1.2. $k_1q_w = 1$. We introduce the parameter $f(\chi, m) = \ln \gamma^{-1} - mq_w$. Obviously f = o(m). Depending on the behavior of the small parameters χ and m^{-1} we have the following cases: a) $f \to +\infty$, b) $f \to k_2$, and c) $f \to -\infty$ (k_2 is some specified constant).

4.1.2. (a) $f \rightarrow +\infty$. This case is entirely analogous to 4.1.1, and everything said about the latter holds.

4.1.2. (b) $f \rightarrow k_2$. The internal expansion is of the form

$$z(\eta; \chi, m) = \exp(-mq_w)z_4(\eta_4) + ...; \eta_4 = \eta \ln \gamma^{-1}$$
(4.3)

In the first approximation Eq. (1.2) is of the form

$$\exp\left(-2k_{2}\right)a_{w}d^{2}z_{4}/d\eta_{4}^{2} = b_{w}z_{4}\left[z_{4}^{2} - \theta_{w}^{*/2}\exp\left(-2k_{1}q_{w}'\eta_{4}\right)\right]$$
(4.4)

. . .

Note that this equation may be also obtained by the formal application to Eq. (1.2) of the method of exponent expansion known in the combustion theory /10/.

We introduce function α using the formula

$$\alpha = -\frac{t'}{r} \left(\frac{a\chi}{b}\right)^{1/2} = \frac{d}{\sqrt{2}} \left(-\frac{dt}{dy}\right)$$

which defines the relation of the local recombination length to the local scale of variation of the quantity t. Note that in the considered limit case $\alpha_w = O(1)$. The canonical form of Eq. (1.2) is

$$d^{2}z_{5} / d\eta_{6}^{2} = z_{5} [z_{5}^{2} - \exp(2 \eta_{5})]$$

$$\eta_{5} = -k_{1}q_{w}'\eta_{4} - \ln \alpha_{w}, z_{5} = \alpha_{w}^{-1}\theta_{w}^{-t/2}z_{4}$$

$$\alpha_{w} = -k_{1}q_{w}'\theta_{w}^{-t/4} (a_{w} / b_{w})^{1/2} \exp(-k_{2})$$

$$(4.5)$$

for which the boundary conditions are

$$\eta_{\mathbf{5}} = -\ln \alpha_{w}, z_{\mathbf{5}} = 0; \ \eta_{\mathbf{5}} \to \infty, \ z_{\mathbf{5}} \sim \exp \eta_{\mathbf{5}} + \exp \left(-\eta_{\mathbf{5}}\right) / 2 + \dots$$

$$(4.6)$$

The last of these conditions is that of merging the internal (4.3) and external (4.1) expansions.

Fig.l

Curves of function $z_{\delta}(\eta_{\delta})$ shown in Fig.1 by solid lines for several values of parameter



 α_w (ln $\alpha_w = 5, 3, 1, -1$) were obtained by exact numerical solution of the nonlinear boundary value problem (4.5), (4.6). The dash line shows the asymptotic behavior of function z_5 as $\eta_5 \rightarrow \infty$ calculated by the second of formulas (4.6).

In input variables we have

$$\begin{aligned} x &= x_{rw} \alpha_w z_{\mathfrak{s}} \left[\left(- \frac{dt}{dy} \right)_w y - \ln \alpha_w \right] \\ \left[\frac{dx}{dy} \right]_w &= \sqrt{2} \alpha_w^2 \psi x_{rw} / d_w, \quad \psi = (\frac{dz_{\mathfrak{s}}}{dy} / \frac{d\eta_{\mathfrak{s}}}{dy})_w \end{aligned}$$

$$\tag{4.7}$$

The curve of function $\psi = \psi \left(lpha_w
ight)$ is also shown in Fig.1, where the dash lines indicate its behavior determined by asymptotic formulas with low and high values of parameter \mathfrak{L}_{n} . It can be shown that these expressions are of the form

$$\psi = 2^{-1/2} \alpha_w^{-2}, \alpha_w \to 0; \ \psi = \sqrt{2} K^2 \left(2^{-1/2} \right) \ln^{-2} \alpha_w, \ \alpha_w \to \infty$$

$$(4.8)$$

where $K(2^{-1/2}) = 1.8541$ is a complete elliptic integral of the first kind in the normal Legendre form /12/.

4.1.2. (c) $f \rightarrow -\infty$. In this case the discontinuity region cannot be defined by a single asymptotic expansion, although it is asymptotically fine. It is necessary to consider two expansions one of which holds in the recombination layer directly adjacent to the wall, and the other in the transition layer between the recombination layer and the chemically stable region. Expansion of the intermediate layer is of the form

 $z(\eta; \chi, m) = \gamma z_{\mathbf{6}}(\eta_{\mathbf{6}}) + ...; \quad \eta_{\mathbf{6}} = (\eta - \delta) \ln \gamma^{-1}$

where δ denotes the root of the algebraic equation $\alpha(\delta) = 1$.

Since $\alpha \to \infty$ as $\eta \to 0$, $\alpha \to 0$ as $\eta \to \infty$, and $\alpha' < 0$, the solution of this equation exists and is unique; it can be shown that $\delta = O(f/m) = o(1)$. Note that the quantity in the right-hand side of this equation may be taken equal not only to unity but to any other constant quantity. The selection of that constant other than unity results only in a shift of the reference point of the η_{6} -coordinate.

In the first approximation Eq. (1.2), after the substitution of variables

$$\eta_{6} = \eta_{7} / (-k_{1}q_{w}'), \quad z_{6} = -k_{1}q_{w}' (a_{w} / b_{w})^{1/2} z_{7}$$

reduces to the canonical form coinciding with (4.5) (except for the substitution of subscript 7 for 5). As $\eta_7 \rightarrow \infty$ the boundary condition for function z_7 is the same as the second of conditions (4.6). The second boundary condition specifies the absence of singularities as $\eta_7 \rightarrow -\infty$.

The curve of function $z_7(\eta_7)$ appears in Fig.l in the form of a dash-dot line. It can be shown that the asymptotics of that function are of the form $z_7 = -\sqrt{2}/\eta_7$ as $\eta_7 \to -\infty$.

In input variable for the molar concentration in the transition layer we have

$$x = x_{rb}z_{7}[V 2(y - y_{b}) / d_{b}]$$
(4.3)

Expansion of the recombination layer is of the form

 $z(\eta; \chi, m) = \sqrt{\chi} \delta^{-1} z_s(\eta_s) + ...; \quad \eta_s = \eta / \delta < 1$

In the first approximation Eq. (1.2) assumes the form

$$a_w d^2 z_8 / d\eta_8^2 = b_w z_8^3 \tag{4.10}$$

whose boundary condition for $\ \eta_8=0$ is the same as the first of conditions (1.3), and the boundary condition as $\eta_{0} \rightarrow 1$ is obtained from the condition of merging with the expansion of the transition layer

$$\eta_{8} = 0, \ z_{8} = 0; \ \eta_{8} \to 1, \ z_{8} \sim (2a_{w} \ / \ b_{w})^{1/2} (1 - \eta_{8})^{-1} + \dots$$
(4.11)

Solving problem (4.10), (4.11) in input variables for the molar concentration in the recombination layer and of the derivative of molar concentration at the wall we obtain

$$x = x_{rb} K \left(2^{-1/2} \right) \frac{d_0}{y_b} \left\{ \frac{2}{1 + \operatorname{cn} \left[2K \left(2^{-1/2} \right) y / y_0, \, 2^{-1/2} \right]} - 1 \right\}^{1/2}$$
(4.12)

$$(dx / dy)_w = K^2 (2^{-i/i}) (d_0 / y_0) x_{r0} / y_0$$
(4.13)

where cn is the Jacobi elliptic function /12/.

4.1.3. $k_1q_w > 1$. As previously indicated, the external expansion (4.1) is applicable when $\eta > \eta_{e}$. The nonuniformity region contains an asymptotically thin transition layer and a recombination layer of thickness $\eta_s = O$ (1). The expansion of the transition layer is of the form $z(\eta; \dot{\chi}, m) = \gamma z_0(\eta_0) + ...; \quad \eta_0 = (\eta - \eta_{s_1}) \ln \gamma^{-1}$

where η_{s_1} denotes the root of the algebraic equation $\alpha(\eta_{s_1}) = 1$; it can be shown that $\eta_{s_1} = \eta_s + o(1)$. The problem for z_9 is analogous to that for function z_7 . Using input variable es we again obtain formula (4.9), if we substitute in the latter subscript s1 for δ .

Expansion of the recombination layer, which is applicable when $\eta < \eta_s$, is of the form $1/\sqrt{n} \pi_{-1}(n) + \dots$

$$z(\eta; \chi, m) = V \chi z_{10}(\eta) + ...$$

For function z_{10} we obtain the problem

(4 9)

$$\begin{array}{l} (d / d\eta) \; (adz_{10} / d\eta) = bz_{10}^{3} \\ \eta = 0, \, z_{10} = 0; \; \eta \to \eta_{s}, \, z_{10} \sim (2a_{s} / b_{s})^{1/2} \; (\eta_{s} - \eta)^{-1} + \dots \end{array}$$

which is generally a nonlinear boundary value problem with variable coefficients, has no analytic solution, and has to be solved numerically.

4.2. $m/\ln\gamma^{-1}\to 0$. In this case the solution of the problem is entirely analogous to that considered in 4.1.1.

4.3. $m / \ln \gamma^{-1} \rightarrow \infty$. Here the solution of the problem substantially depends on the character of damping of function $q(\eta)$ as $\eta \rightarrow \infty$. Here, the analysis is restricted to the case of exponential damping

$$\eta \rightarrow \infty$$
, $q(\eta) \sim k_3 \exp(-k_4 \eta) + \dots$

where k_3 and k_4 are arbitrary positive constants.

We denote by ω the root of the equation $\alpha(\omega) = 1$. It can be shown that $\omega^{-1} = o(1)$. We assume for simplicity that the expression

$$m \left[q \left(\eta\right) - k_3 \exp\left(-k_4 \eta\right)\right]$$

approaches at the limit $\chi \to 0$, $m^{-1} \to 0$ (with $(\eta_{11} = \eta / \omega \text{ fixed})$ zero. For ω we can obtain the following expression:

$$\omega = \frac{1}{k_4} \ln \frac{mk_3}{\ln \gamma^{-1}} + \frac{\ln k_4}{k_4 \ln \gamma^{-1}} [1 + o(1)]$$

The external expansion

 $z(\eta; \chi, m) = \exp \{\ln (k_4 \gamma) \exp [k_4 \omega (1 - \eta_{11})]\} + \dots$

can be calculated directly. This expansion is valid for $\eta_{11}>1.$ Expansion of the transition layer is of the form

$$z (\eta; \chi, m) = \gamma z_{12} (\eta_{12}) + ...; \qquad \eta_{12} = (\eta - \omega) \ln \gamma^{-1}$$

The problem for z_{12} is analogous to that for function z_7 . Hence

$$z_{12} = k_4 z_7 (k_4 \eta_{12})$$

Using the input variables for the molar concentration in the transition layer we again obtain formula (4.9) by substituting in the latter subscript ω for δ .

Expansion for the recombination layer, valid when $\eta_{11} < 1$, is of the form

$$z(\eta; \chi, m) = \sqrt{\chi \omega^{-1} z_{13}(\eta_{11}) + \dots}$$
(4.14)

In the first approximation Eq. (1.2) assumes the form

$$d^2 z_{13} / d\eta_{11}^2 = z_{13}^3 \tag{4.15}$$

whose solution that satisfies the condition of merging as $\eta_{11} \rightarrow 1$ is zero when $\eta_{11} = 0$, and in input variables is again given by formula (4.12) in which subscript ω is substituted for δ .

Note that when $\eta_{11} = O(\omega^{-1})$ functions *a* and *b* generally differ from unity, and Eq. (4.15) is no longer a good approximation of Eq. (1.2). Thus there exists between the recombination layer and the wall in this case a transition region (chemically frozen layer) of thickness O(1). A solution valid in that region is of the form

$$z(\eta; \chi, m) = \sqrt{\chi} \omega^{-2} z_{14}(\eta) + \dots \qquad (4.16)$$

For function z_{14} we obtain the boundary value problem

$$(d / d\eta) (a dz_{14} / d\eta) = 0$$

$$\eta = 0, z_{14} = 0; \eta \to \infty, z_{14} \sim \sqrt{2}K^2 \ (2^{-1/2}) \ \eta + ...$$

(the last condition relates to the merging of expansions (4.14) and (4.16)).

Solving the problem (4.17) for molar concentration in the frozen layer and for the derivative of molar concentration at the wall in input variables, we obtain

$$x = x_{r\omega} K^2 (2^{-1/z}) \frac{d_{\omega}}{y_{\omega}} \frac{1}{y_{\omega}} \int_{0}^{y} \frac{dy}{a}, \ \left(\frac{dx}{dy}\right)_{\omega} = K^2 (2^{-1/z}) \frac{d_{\omega}}{y_{\omega}} \frac{1}{a_w} \frac{x_{r\omega}}{y_{\omega}}$$

5. Discussion of results. The asymptotic solutions obtained in Sects.3 and 4 have a clear physical meaning. When the wall temperature is so high that the respective local recombination length is considerably smaller than the characteristic scale of the function variation t (i.e. parameter α_w is considerably less than unity), a layer is formed near the wall, where transition from the chemically stable concentration of charged particles at the wall temperature at the external boundary of the layer to zero concentration on the wall takes place. The chemically stable charged particle concentration in the layer is in the first approximation stable, since the scale of that concentration variation is the same as that of variation of funtion t. The charged particle concentration distribution in the transition layer is determined by function th. The terms that define ionization, recombination, and diffusion in the layer are of the same order of magnitude. The thickness of the layer is equal several local

(4 3.5.)

recombination lengths.

As the wall temperature decreases the respective chemically stable concentration decreases, while the local recombination length increases. As that length becomes comparable to the characteristic scale of variation of function t near the surface (i.e. parameter α_w becomes comparable to unity), the chemically stable concentration of charged particles in the transition layer can no longer be assumed constant, and the structure of the layer changes. The concentration distribution of charged particles in the layer is defined by function z_5 which parametrically depends on α_w .

With further lowering of the wall temperature the transition layer thickness continues to increase, and when parameter α_w markedly exceeds unity, the transition layer moves away from the wall in whose immediate vicinity a recombination layer is formed. The recombination layer is now in the neighborhood of point y_{s1} , where the local recombination length is equal to the local scale of the chemically stable charged particle concentration. From the physical point of view this condition is fully understandable, since when the local recombination length becomes comparable to the smallest of the characteristic local scales, there is no reason for expecting equilibrium. The charged particle concentration distribution in the transition layer is defined by the universal function z_7 . As in previous cases, the transition layer thickness is equal to the thickness of several local recombination lengths, and is considerably smaller than the characteristic scale of gas temperature variation. The terms that define ionization, recombination, and diffusion are in that layer of the same order of magnitude.

Since in the recombination layer the term which defines ionization is small, it can be omitted in the first approximation. The recombination layer thickness is determined by the position of point y_{i1} and, depending on parameters χ and m and the gas temperature distribution, can be small, large (compared to the characteristic scale of gas temperature variation), or be of order unity. In the first two cases the equation of the recombination layer can be solved analytically for any arbitrary dependence of coefficients of the ambipolar diffusion and recombination on the gas temperature. In the third case it is generally necessary to resort to numerical methods for solving this equation.

When the recombination layer thickness considerably exceeds the characteristic scale of the gas temperature variation it is, generally, necessary to introduce in the analysis the chemically frozen layer of thickness of the order of the characteristic scale of gas temperature variation, directly adhereing to the wall. The terms that define ionization and recombination are small and can be omitted in the first approximation. The obtained equation can be solved analytically for any arbitrary dependence of the ambipolar diffusion coefficient on temperature.

It is important to note that the asymptotic solutions obtained for various limit cases are in agreement when passing to limit from one to another. Thus the dependence of the derivative of molar concentration of charged particles at the wall on parameter α_{w} , which is defined by formula (4.7) at transition to limits $\alpha_{w} \rightarrow 0$ and $\alpha_{w} \rightarrow \infty$ with allowance for formulas (4.8) is in agreement with formulas (3.5) and (4.13).

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